

# OPTIMIZATION OF CASP-CUSUM SCHEMES BASED ON TRUNCATED LOG-LOGISTIC DISTRIBUTION USING LOBATTO INTEGRATION METHOD

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## Abstract

Among several methods to control the quality and reliability, Acceptance Sampling Plans are introduced to accept or reject the lots of finished products. Such types of methods are involved in the inspection quality of product which is destructive where 100% inspection is not possible such as bullets, batteries, bulbs and so on. This paper study the CASP-CUSUM Schemes based on the assumption that the continuous variable under consideration follows a truncated Log logistic distribution. It is used the Lobatto Integration Method to solve the truncated integral equations. The Log logistic Distribution plays a vital role in Statistical Quality Control, particularly in estimating reliability by considering its distribution. Optimum CASP-CUSUM Schemes are suggested based on numerical results obtained.

**Keywords:** CASP-CUSUM Schemes, type-C OC Curve, ARL, Truncated Log logistic distribution

## 1. Introduction

Acceptance sampling is concerned with inspection and decision making regarding lots of product by the techniques of quality of assurance in 1930's and 1940's Acceptance Sampling Plans were one of the important statistical tools on the field of Statistical Quality Control. These techniques require specification of a probability model describing the life of the products. In this process the Log- logistic distribution has been studied by Shah and Dave [08].

Acceptance Sampling Plans are widely used in business, industry, producing engineering products etc. For example a company receives a segment of product from a vender, the product is after a component or raw material used in the manufacturing process. A sample is taken from a lot and examined the quality with respect to certain characteristics of interest. On the basis of information supplied by the sample, an optimal decision is to be made. Often a decision is taken to accept the lot or reject the lot. After the decision, accepted lots are put it into production while rejected lots may be returned to the vendor. In such case, to take optimal decisions with regard to "Acceptance Sampling Plans" are widely used from past two decades. Thus most acceptance sampling plans are working as an alarm in decision making therefore we will develop a procedure for calculating the values relevant characters  $P(A)$ , ARL's of CASP-CUSUM to make the required decision.

In these lines Sobel and Tischendorf [09] studied acceptance sampling plans based on an exponential distributions. Gupta and Groll [03] developed sampling plans based on Gamma life test sample data. Goode and

Kao [02] proposed samples plans based on the Weibull distribution and determined appropriate measures of CUSUM schemes.

In this study, we proposed acceptance sampling method based on the truncated Log-logistic distribution under the assumption that the variable under study distributed according to truncated Log-logistic distribution under this assumption we determined an appropriate measures for CUSUM schemes by using Lobatto Method of Integration. Thus it is more worthwhile to study some interesting characteristics of this distribution. The characteristics and applications of the Log-logistic distribution were discussed.

## 2. Log-Logistic Distribution

A continuous random variable X assuming non-negative values is said to have Log-logistic Distribution with parameter  $\beta, \alpha > 0$ , if its probability density function is given by:

$$f(x, \alpha, \beta) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{(1+(x/\alpha)^\beta)^2} \quad (2.1)$$

## 3. Truncated Log-logistic Distribution

The random variable X is said to follow a Truncated Log-logistic Distribution if:

$$f_B(x) = \frac{\left[ \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{(1+(x/\alpha)^\beta)^2} \right]}{1 - \left[ \frac{1}{1 + \left(\frac{B}{\alpha}\right)^\beta} \right]} \quad \alpha, \beta. > 0, B > 0, X \leq B \quad (3.1)$$

Where ,B is the upper truncation point of the Log-logistic distribution.

## 4. Description of the Plan and Type of OC curve

Beattie D.W [01] has suggested the method for constructing the continuous acceptance sampling plan. The procedure, suggested by him consists of a chosen decision interval namely, “**Return interval**” with the length  $h'$ , above the decision line is taken. We plot on the chart the sum  $S_m = \sum (X_i - k)$ ,  $X_i$ 's ( $i = 1, 2, 3, \dots$ ) are distributed independently and k is the reference value. If the sum lies in the area of normal chart, the product is accepted and if it lies in the of the return chart, the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum, i.e.,  $h + h'$ .
2. When the decision line is reached or crossed from above, the next point on the chart is to be plotted from the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure in brief is given below:

1. Start plotting the CUSUM at 0.
2. The product is accepted when  $S_m = \sum (X_i - k) < h$ ; when  $S_m < 0$ , return cumulation to 0.
3. When  $h < S_m < h + h'$  the product is rejected: when  $S_m$  crosses  $h$ , i.e., when  $S_m > h + h'$  and continue rejecting product until  $S_m > h + h'$  return cumulation to  $h + h'$ .

The Type – C, OC function which is defined as the probability of acceptance of an item as a function of incoming quality, when sampling rate is same in acceptance and rejection regions. Then the probability of acceptance  $P(A)$  is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \quad (4.1)$$

Where  $L(0)$  = Average Run Length in acceptance zone and

$L'(0)$  = Average Run Length in rejection zone.

Page, E.S. [07] has introduced the formulae for  $L(0)$  and  $L'(0)$  as:

$$L(0) = \frac{N(0)}{1 - P(0)} \quad (4.2)$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \quad (4.3)$$

Where  $P(0)$  == Probability for the test starting from zero on the normal chart,

$N(0)$  = ASN for the test starting from zero on the normal chart,

$P'(0)$  = Probability for the test on the return chart and

$N'(0)$  = ASN for the test on the return chart.

He further obtained integral equations for the quantities

$P(0)$ ,  $N(0)$ ,  $P'(0)$ ,  $N'(0)$  as follows :

$$P(z) = F(k - z) + \int_0^h p(y) f(y + k - z) dy, \dots \tag{4.4}$$

$$N(z) = 1 + \int_0^h N(y) f(y + k - z) dy, \dots \tag{4.5}$$

$$P'(z) = \int_{k+z}^B f(y) dy + \int_0^h P'(y) f(-y + k + z) dy \dots \tag{4.6}$$

$$\left. \begin{aligned} N'(z) &= 1 + \int_0^h N'(y) f(-y + k + z) dy, \\ F(x) &= 1 + \int_A^h f(x) dx : \\ F(k - z) &= 1 + \int_A^{k-z} f(y) dy, \end{aligned} \right\} \dots \tag{4.7}$$

and  $z$  is the distance of the starting of the test in the normal chart from zero.

### 5. Method of Solution

In this section, the procedure of the method of solution studied by Narayana Murthy et al [06] was considered to evaluate the integral equation (4.4) in Narayana Murthy et al [06 ]

The main objective of this paper is to obtain solutions of ARL and Type- C OC curves for various CASP-CUSUM charts by assuming that the  $X_i$ 's follows truncated Log-logistic distribution

There are several methods to solve integral equations given from (4.4) to (4.7). Here we have considered a method suggested by Krishna Murthy, E.B. Sen, S.K. [05] to obtain the numerical solution for P (A). The method briefly as follows:

We first express the integral equation as:

$$F(x) = Q(x) + \int_c^d R(x,t) F(t) dt \tag{5.1}$$

Where  $F(x) = P(z)$

$Q(x) = F(k - z)$

$$R(x, t) = f(y + k - z)$$

Let the integral  $I = \int_c^d f(x) dx$  be transformed to

$$I = \frac{d-c}{2} \int_c^d f(y) dy = \frac{d-c}{2} \sum a_i f(t_i) \tag{5.2}$$

Where  $y = \frac{2x - (c + d)}{d - c}$  where  $a_i$ 's and  $t_i$  's respectively the weight factor and abscissa for the Gauss-

Chebyshev polynomial given M.K. Jain and et al [04] using (4.1) and (4.2), (4.4) can be written as

$$F(x) = Q(x) + \frac{d-c}{2} \sum a_i R(x, t_i) F(t_i) \tag{5.3}$$

Since equation (5.3) should be valid for all values of  $x$  in the interval  $(c, d)$ , it must be true for  $x = t_i, i = 0(1)n$  then obtain

$$F(t_i) = Q(t_i) + \frac{d-c}{2} \sum a_i R(t_j, t_i) F(t_j) \tag{5.4}$$

$$j = 0(1)n$$

Substituting

$F(t_i) = F_i, Q(t_i) = Q_i, i = 0(1)n$ , in (2.4.4), we get

$$F_0 = Q_0 + \frac{d-c}{2} [a_0 R(t_0, t_0) F_0 + a_1 R(t_0, t_1) F_1 + \dots a_n R(t_0, t_n) F_n]$$

$$F_1 = Q_1 + \frac{d-c}{2} [a_0 R(t_1, t_0) F_0 + a_1 R(t_1, t_1) F_1 + \dots a_n R(t_1, t_n) F_n]$$

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$$F_n = Q_n + \frac{d-c}{2} [a_0 R(t_n, t_0) F_0 + a_1 R(t_n, t_1) F_1 + \dots a_n R(t_n, t_n) F_n] \tag{5.5}$$

In the system of equations except  $F_i, i = 0, 1, \dots, n$  are known and hence can be solved for  $F_i$ . We solve the system of equations by the method of iteration. For this we write the system (4.5) as

$$\begin{aligned}
 [1 - Ta_0R(t_0, t_0)]F_0 &= Q_0 + T[a_0R(t_0, t_0)F_0 + a_1R(t_0, t_1)F_1 + \dots a_nR(t_0, t_n)F_n] \\
 [1 - Ta_1R(t_1, t_1)]F_1 &= Q_1 + T[a_0R(t_1, t_0)F_0 + a_1R(t_1, t_1)F_1 + \dots a_nR(t_1, t_n)F_n] \\
 \dots\dots\dots & \dots\dots\dots \\
 \dots\dots\dots & \dots\dots\dots \\
 [1 - Ta_nR(t_n, t_n)]F_n &= Q_n + T[a_0R(t_n, t_0)F_0 + a_1R(t_n, t_1)F_1 + \dots a_nR(t_n, t_n)F_n] \tag{5.6}
 \end{aligned}$$

Where  $T = \frac{d - c}{2}$ .

To start the Iteration process, let us put  $F_1 = F_2 = \dots F_n = 0$  in the first equation of (5.1), we then obtain a rough value of  $F_0$ . Putting this value of  $F_0$  and  $F_2 = F_3 \dots F_n = 0$  in the second equation, we get a rough value  $F_1$  and so on. This gives the first set of values  $F_i, i = 0, 1, 2, \dots, n$  which are just the refined values of  $F_i, i = 0, 1, 2, \dots, n$ . The process is continued until two consecutive sets of values are obtained up to a certain degree of accuracy. In the similar way solutions  $P'(0), N(0), N'(0)$  can be obtained.

Now we proceed to obtain the results of  $P(A)$  for various parametric values in the following section.

**6. COMPUTATION OF P (A) AND ARL's**

In this Section CASP-CUSUM schemes are obtained when the variable considered follows a Truncated Log-logistic distribution and hence (3.1) is to be substituted in equation (2.1) from (4.4) through (4.7). There are several methods to solve integral equations (4.4) to (4.7) when the variable under study distributed an appropriate truncated probability distribution. Therefore in this paper we consider method – I developed by Krishna Murthy and Sen S.K.,[05] to obtain the numerical solutions for  $P(A)$  and ARL's for CASP-CUSUM Schemes when  $f(.)$  is truncated Log-logistic probability density function (see equation (3.1)) for various parameters like

- i)  $h$  = Size of the acceptance zone
- ii)  $h'$  = Size of the rejection zone
- iii)  $\alpha, \beta$  = Parameters of the distribution
- iv)  $B$  = Truncated point of the random variable  $X$  and
- v)  $k$  = The reference value, are calculated through the computer programmes.

## Numerical Results and Conclusions

The calculated values are given in the following tables.

TABLE - 6.1

VALUES OF ARLs AND TYPE –C OC CURVES when

$$\beta = 2 \quad \alpha = 0.2 \quad k = 2 \quad h = 0.25 \quad h' = 0.25$$

B	L(0)	L'(0)	P(A)
2.8	152.93988	1.2897642	0.9916373
2.7	163.03485	1.2897843	0.9921510
2.6	176.02032	1.2898067	0.9927257
2.5	193.30388	1.2898316	0.9933717
2.4	217.37144	1.2898597	0.9941011
2.3	253.08168	1.2898912	0.9949291
2.2	311.36914	1.2899269	0.9958743
2.1	423.07697	1.2899675	0.9969603
2.0	721.90216	1.2900139	0.9982162
1.9	4030.51514	1.2900672	0.9996800

TABLE - 6.2

VALUES OF ARLs AND TYPE –C OC CURVES when

$$\beta = 2 \quad \alpha = 0.2 \quad k = 2 \quad h = 0.5 \quad h' = 0.5$$

B	L(0)	L'(0)	P(A)
2.6	94.78883	1.2823602	0.9866520
2.5	99.87975	1.2821832	0.9873254
2.4	106.32011	1.2819833	0.9880859
2.3	114.70221	1.2817566	0.9889488
2.2	126.02596	1.2814981	0.9899338
2.1	142.10959	1.2812012	0.9910650
2.0	166.65361	1.2808582	0.9923729
1.9	208.50887	1.2804587	0.9938964
1.8	295.45215	1.2799897	0.9956864
1.7	582.59613	1.2794341	0.9978087

TABLE - 6.3

VALUES OF ARLs AND TYPE –C OC CURVES when

$$\beta = 2 \quad \alpha = 0.2 \quad k = 2 \quad h = 0.75 \quad h' = 0.75$$

B	L(0)	L'(0)	P(A)
2.3	56.50605	1.1410203	0.9802068
2.2	59.27536	1.1404010	0.9811241
2.1	62.80463	1.1396896	0.9821768
2.0	67.43871	1.1388665	0.9833930
1.9	73.76637	1.1379073	0.9848085
1.8	82.87852	1.1367794	0.9864694
1.7	97.05300	1.1354412	0.9884361
1.6	121.96008	1.1338358	0.9907889
1.5	176.72929	1.1318870	0.9936361
1.4	392.07733	1.1294874	0.9971275

TABLE - 6.4

VALUES OF ARLs AND TYPE –C OC CURVES when

$$\beta = 2 \quad \alpha = 0.2 \quad k = 2 \quad h = 1 \quad h' = 1$$

B	L(0)	L'(0)	P(A)
2.0	33.93274	0.7352369	0.9787920
1.9	35.54634	0.7322696	0.9798154
1.8	37.64441	0.7287928	0.9810078
1.7	40.46931	0.7246824	0.9824080
1.6	44.45396	0.7197749	0.9840665
1.5	50.45522	0.7138512	0.9860492
1.4	60.43930	0.7066109	0.9884439
1.3	80.12746	0.6976362	0.9913686
1.2	136.14742	0.6863309	0.9949842
1.1	1374.02063	0.6718251	0.9995113



TABLE - 6.5

VALUES OF ARLs AND TYPE –C OC CURVES when

$$\beta = 2 \quad \alpha = 0.2 \quad k = 4 \quad h = 0.25 \quad h' = 0.25$$

B	L(0)	L'(0)	P(A)
4.2	624.93048	1.2896136	0.9979407
4.1	674.96918	1.2896197	0.9980930
4.0	738.64679	1.2896262	0.9982571
3.9	822.20032	1.2896332	0.9984339
3.8	936.67578	1.2896407	0.9986250
3.7	1102.83740	1.2896488	0.9988320
3.6	1365.58057	1.2896576	0.9990565
3.5	1842.67188	1.2896671	0.9993006
3.4	2975.78027	1.2896776	0.9995668
3.3	9059.20313	1.2896889	0.9998577

TABLE - 6.6

VALUES OF ARLs AND TYPE –C OC CURVES when

$$\beta = 2 \quad \alpha = 0.2 \quad k = 4 \quad h = 0.5 \quad h' = 0.5$$

B	L(0)	L'(0)	P(A)
3.2	233.55492	1.2830954	0.9945363
3.1	250.36848	1.2830019	0.9949017
3.0	271.93246	1.2828989	0.9953045
2.9	300.55060	1.2827849	0.9957500
2.8	340.26788	1.2826585	0.9962446
2.7	398.96600	1.2825176	0.9967957
2.6	494.25485	1.2823602	0.9974122
2.5	675.29822	1.2821832	0.9981049
2.4	1150.61890	1.2819833	0.9988871
2.3	5690.04883	1.2817566	0.9997748

At the hypothetical values of the parameters  $\alpha, \beta, k, h$  and  $h'$  given at the top of each table, we computed optimum truncated point B at which P (A) the probability of accepting an item is maximum and also

attained ARL's values which represents the acceptance zone  $L(0)$  and rejection zone  $L'(0)$  values. The values of truncated point B of random variable X,  $L(0)$ ,  $L'(0)$  and the values for Type – C OC Curve, i.e. P (A) were given in columns I, II, III and IV respectively.

From Tables 6.1 to 6.6 we made the following conclusions

- By go through the above tables the values of h, h' increases then related values of P (A) decreases. So the sizes of accepted and rejected zones and P (A) are inversely related.
- By examining the above table we found that the value of parameter of Log-Logistic distribution increases then P (A) is also increases. So, the parameter of Log-Logistic distribution and P (A) are positively related.
- By in sighting the above tables the reference value of K increase then L (0) increases. So, reference value and L (0) are positively related.
- By observing the above tables the values of h, h' increases then related values of L (0) decreases. So the sizes of accepted and rejected zones and L (0) are inversely related.
- From Table 6.1 to 6.6 it is observed that the size of the acceptance zone is increases as the reference value K increases.
- By observing the above tables the reference value of K increase then P (A) will also increases. So, reference value and P (A) are positively related.

Table No 6. 7

#### Optimum Values of CASP-CUSUM Schemes

B	$\alpha$	$\beta$	k	h	$h'$	L (0)	$L'(0)$	P (A)
1.9	0.2	2	2	0.25	0.25	4030.5151	1.2900672	0.9996800
1.7	0.2	2	2	0.5	0.5	582.5961	1.2794341	0.9978087
1.4	0.2	2	2	0.75	0.75	392.0773	1.1294874	0.9971275
1.1	0.2	2	2	1	1	1374.0206	0.6718251	0.9995113
<b>3.3</b>	<b>0.2</b>	<b>2</b>	<b>4</b>	<b>0.25</b>	<b>0.25</b>	<b>9059.2031</b>	<b>1.2896889</b>	<b>0.9998577</b>
2.3	0.2	2	4	0.5	0.5	5690.0488	1.2817566	0.9997748

By observing the above table the values of L (0) and P (A) reaches their maximum i.e. **9059.2031**, **0.9998577** respectively. We obtain optimum CASP-CUSUM Scheme as

$$\left[ \begin{array}{l} B = 3.3 \\ \alpha = 0.2 \\ \beta = 2 \\ k = 4 \\ h = 0.25 \\ h' = 0.25 \end{array} \right]$$

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