EFFECT OF VOLUME FRACTION ON NON-NEWTONIAN FLOW OF DUSTY FLUID BETWEEN TWO OSCILLATING PARALLEL PLATES

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Abstract: In this paper the isothermal flow of dusty viscous incompressible conducting between two parallel plates is studied. The effect of various parameters associated with the flow problem have been analysed with the help of graphs. These results show that fluid velocity and particle velocity are effected by magnetic field for different values of volume fraction and non Newtonian factor.

Keywords: Incompressible fluid, skin fraction, magnetic field, non Newtonian factor, volume fraction.

Introduction: The study of heat and mass transfer in particular flows is emerging as an important topic of research. The study is very useful in scientific and technological applications such as sedimentation, purification of water and in petroleum industry etc.

The field is so important that number of scientists is motivated for research on the topic Dubey, and Srivastava (1972) investigated unsteady flow of dusty viscous flow with uniform distribution of dust particle in a channel bounded by two parallel flat plat. Prasad and Ramacharyulu N.C.P. studied unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient. Deb Nath and Ghosh (1989) studied unsteady hydro magnetic flows of a dusty viscous fluid between two oscillating plates. Sharma and Mathur (1995) investigated Steady Laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source/sink. Suraju Olusegun Azadi (2005) studied unsteady flow of dusty viscous fluid between two parallel plates.

Hazem Attia (2005) investigated the topic of unsteady flow of dusty conducting fluid between parallel porous plates with temperature dependent viscosity.


Saffman (1962) derived the equation describing the motion of gas carrying small dust particle and equations satisfied by small disturbances of a steady laminar flow.

Ganguly and Lehri studied the motion of an isothermal dusty viscous incompressible fluid between two infinite parallel plates where both plates are considered to be oscillating harmonically with different amplitudes and frequency in their own planes.

In another application Haldar investigated the flow of blood through a constricted artery in the presence of an external transverse magnetic field using Adomian’s decomposition method. The expression for the two term approximation to the
solution of stream function, axial velocity component and wall shear stress and obtained in this analysis.

The present paper considers the motion of an isothermal conducting dusty viscous incompressible fluid between two infinite parallel plates under the influence of gravitational precipitation, using analytical solutions.

**Formulation of problem**

Let us consider two dimensional incompressible plane viscous flow between two parallel plates with distance $d$ apart. We consider axial flow of fluid between the plates, as the plates are very wide and very long. For an isothermal flow the equations of motion of an unsteady dusty viscous incompressible fluid under gravity are

\[ (1 - \varphi) \frac{\partial U'}{\partial t} + (U', \nabla) U' = -\frac{1}{\rho} \nabla p - \frac{\sigma B_0^2}{\rho (1+c^2)} + v \nabla^2 U' + \frac{K_N}{\rho} (V' - U') + g' \]

\[ \nabla U' = 0 \]  
\[ (U', \nabla) V' = g' + \frac{K}{m} (U' - V') \]

\[ \frac{\partial \rho}{\partial t} + \nabla (\rho \nabla U') = 0 \]  
\[ \frac{\partial \rho}{\partial t} + \nabla (\rho \nabla V') = 0 \]

Where $U'$ and $V'$ are the local velocities of fluid and dust particles,

\[ \rho \] : Density of fluid,
\[ p \] : Static fluid pressure,
\[ v \] : Kinetic viscosity,
\[ N \] : Number of dust particles per unit volume
\[ K \] : Resistances coefficient ($K = 6\pi \mu a$)
\[ \varphi \] : Volume fraction of dust particle
\[ c \] : Non-Newtonian factor,
\[ B_0 \] : Constant external magnetic field parameter,
\[ g' \] : Acceleration due to gravity and
\[ m' \] : Mass of the particles.

From the equation of continuity we have

\[ \nabla U' = 0 \quad i.e.: \frac{\partial U'}{\partial x'} = 0 \quad U' = U'(y') \]

\[ \nabla V' = 0 \quad i.e.: \frac{\partial V'}{\partial x'} = 0 \quad V' = V'(y') \]

So equation (1) and (3) becomes

\[ (1 - \varphi) \frac{\partial U'}{\partial t} + (U', \nabla) U' = -\frac{1}{\rho} \nabla p - \frac{\sigma B_0^2}{\rho (1+c^2)} + \frac{v}{\rho} \nabla^2 U' + \frac{K_N}{\rho} (V' - U') + g' \]

\[ \frac{\partial \rho}{\partial t} = g' + \frac{K}{m} (U' - V') \]

**Case I: Oscillating parallel plates**

In this case we consider the parallel plates are oscillating. So let us consider the lower plate is oscillating at an amplitude $a_1$ and with frequency $\alpha_1$, and the upper plate oscillates at an amplitude $a_2$ with frequency $\alpha_2$. Moreover we consider that the flow has a negligible convection, have constant pressure gradient $N$. The boundary conditions in this case are

\[ U' = a_1 e^{-i\alpha_1 t} \quad \text{at} \quad y' = 0 \]

And

\[ U' = a_2 e^{-i\alpha_2 t} \quad \text{at} \quad y' = d \]

Now eliminating $v$ between (7) and (8) we get

\[ (1 - \varphi) U''_{tt} = V'' U''_{yyt} + \frac{K N}{\rho} U''_{yy} - \left\{ (1 - \varphi) \frac{K}{m} + \frac{\sigma B_0^2}{\rho (1+c^2)} + \frac{K N}{\rho} \right\} U''_t - \frac{K \sigma B_0^2}{\rho (1+c^2)} \]

\[ \frac{\partial \rho}{\partial t} = (\rho + m N) g' \], so equation (10) becomes

\[ (1 - \varphi) U''_{tt} = V'' U''_{yyt} + \frac{K N}{\rho} U''_{yy} - \left\{ (1 - \varphi) \frac{K}{m} + \frac{\sigma B_0^2}{\rho (1+c^2)} + \frac{K N}{\rho} \right\} U''_t - \frac{K \sigma B_0^2}{\rho (1+c^2)} \]

Now from the boundary conditions we seek the solution as

\[ U' = a_1 f(y') e^{-i\alpha_1 t} + a_2 g(y') e^{-i\alpha_2 t} \]

Consider $A = \frac{K N}{m}$,

\[ B = (1 - \varphi) \frac{K}{m} + \frac{\sigma B_0^2}{\rho (1+c^2)} + \frac{K N}{\rho} \] And

\[ C = \frac{K \sigma B_0^2}{\rho (1+c^2)} \]
So equation (11) becomes

\[(1 - \varphi)U'' + V'U_{yy} + AU''_{yy} - BU'_{x} - CU' = 0\]

Now substituting the values of \(U''_{xx}, U_{yy}, U''_{yy}, U'_{x}\) and \(U'\) in (12) we get

\[f'' + Pf = 0\]

Where \(P = \frac{(1 - \varphi)\lambda^{2} + iB\lambda_{2} - C}{A - iv\lambda_{1}}\)

Similarly we can write

\[g'' + Qg = 0\]

Where \(Q = \frac{(1 - \varphi)\lambda^{2} + iB\lambda_{2} - C}{A - iv\lambda_{2}}\)

Now the solution of (14) can be written as

\[f(y) = C_{0} \cos Py + C_{1} \sin Py\]

With boundary conditions

\[f(y) = 1 \quad \text{and} \quad f(d) = 0\]

Using these boundary conditions we get

\[f(y) = \frac{\sin Pd}{\sin Pd}\]

(16)

Similarly the solution of (15) can be written as

\[g(y) = C_{2} \cos Qy + C_{3} \sin Qy\]

With boundary conditions

\[g(0) = 0 \quad \text{and} \quad g(d) = 1\]

Using these we get

\[g(y) = \frac{\sin Qy}{\sin Qy}\]

(17)

Using (16) and (17) in (12) we get

\[U' = \frac{a_{1} \sin P(d - y)}{\sin Pd} e^{-i\lambda_{1}t} + \frac{a_{2} \sin Qy}{\sin Qy} e^{-i\lambda_{2}t}\]

(18)

Substituting (18) in (7) we get

\[V' = \frac{ma'}{K} + \left(1 - \frac{i\lambda_{1}p(1 - \varphi)}{KN} + \frac{\nu p^{2} \rho}{KN} + \frac{\sigma B_{0}^{2}}{(1 - e^{2})KN} \right) \frac{a_{1} \sin P(d - y)}{\sin Pd} e^{-i\lambda_{1}t} + \left(1 - \frac{i\lambda_{2}p(1 - \varphi)}{KN} + \frac{\nu p^{2} \rho}{KN} + \frac{\sigma B_{0}^{2}}{(1 + e^{2})KN} \right) \frac{a_{2} \sin Qy}{\sin Qy} e^{-i\lambda_{2}t}\]

(19)

Case II: Fixed upper plates

In this case we consider that the upper plate is fixed and the lower plate executes simple harmonic motion in its own plane. So we have

\[a_{1} = 0 \quad \text{and} \quad \lambda_{1} = 0\]

So the velocity of the fluid becomes

\[U' = \frac{\alpha_{1} \sin P(d - y)}{\sin Pd} e^{-i\lambda_{1}t}\]

(20)

And that of dust particle becomes

\[V' = \frac{ma'}{K} + \left(1 - \frac{i\lambda_{1}p(1 - \varphi)}{KN} + \frac{\nu p^{2} \rho}{KN} + \frac{\sigma B_{0}^{2}}{(1 - e^{2})KN} \right) \frac{a_{1} \sin P(d - y)}{\sin Pd} e^{-i\lambda_{1}t}\]

(21)

Case III: Both plates oscillating with same amplitude and frequency

In this we consider the case where both the plates oscillates at an amplitude \(a\) and frequency \(\lambda\).

So we have \(a_{1} = a_{2} = a\) and \(\lambda_{1} = \lambda_{2} = \lambda\) and \(P = Q = R\)

\[U' = \frac{a \sin R(d - y)}{\sin Rd} e^{-i\lambda t} + \frac{a \sin Ry}{\sin Rd} e^{-i\lambda t}\]

Or

\[U' = \frac{a \cos R(d - y)}{\cos Rd} e^{-i\lambda t}\]

(21)

And

\[V' = \frac{ma'}{K} + \left(1 - \frac{i\lambda_{1}p(1 - \varphi)}{KN} + \frac{\nu p^{2} \rho}{KN} + \frac{\sigma B_{0}^{2}}{(1 + e^{2})KN} \right) \frac{a_{1} \sin P(d - y)}{\sin Pd} e^{-i\lambda_{1}t}\]

(22)
Table I: Magnetic field ($B_0$) and velocity of fluid ($U'$)

\[ K = 0.001, \nu = 0.01, m = 0.001, \sigma = 0.01, \rho = 0.01, a_1 = 0.3, a_2 = 0.4, \varphi = 0.01, \]
\[ N = 0.1, \lambda_1 = 0.01, \lambda_2 = 0.02, d = 2, y = 1.1, t = 1 \text{ g}' = 1 \]

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<tr>
<th>$B_0$</th>
<th>$U'(\text{absolute})$</th>
<th>$U'(\text{absolute})$</th>
<th>$U'(\text{absolute})$</th>
<th>$U'(\text{absolute})$</th>
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<td>(non Newtonian factor) $c = 0.09$</td>
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Table II: Magnetic field ($B_0$) and velocity of Dust particle ($V'$)

\[ K = 0.001, \nu = 0.01, m = 0.001, \sigma = 0.01, \rho = 0.01, a_1 = 0.3, a_2 = 0.4, \varphi = 0.01, \]
\[ N = 0.1, \lambda_1 = 0.01, \lambda_2 = 0.02, d = 2, y = 1.1, t = 1 \text{ g}' = 1 \]

<table>
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<tr>
<th>$B_0$</th>
<th>$V'(\text{absolute})$</th>
<th>$V'(\text{absolute})$</th>
<th>$V'(\text{absolute})$</th>
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</table>
Figure I: Magnetic field ($B_0$) and velocity of fluid ($U'$)

![Magnetic field vs Velocity of fluid](image1)

Figure II: Magnetic field ($B_0$) and velocity of Dust particle ($V'$)

![Magnetic field vs Velocity of dust particle](image2)
Results and discussion

The above graph shows the variation in velocity of the fluid and the particle with change in the magnetic field parameter under the effect of volume fraction of the dust.

Figure I depict the change in the velocity of the fluid with changing magnetic field parameter. It can be shown from the graph that on increasing the value of magnetic field parameter the graph of the velocity takes wavy form. This is also clear from the expression of velocity of fluid obtained mathematically. The graph takes the resonance character having the same amplitude till the end. It has also been observed that on increasing the value of volume fraction of the dust the initial velocity also increases.

Figure II shows the change in the velocity of the dust with changing magnetic field of the system. It is evident mathematically and graphically that on increasing the magnetic effect the velocity takes the form of the wave. The waves so formed have very small amplitude in the beginning then the amplitude of the wave goes on increasing and finally becomes maximum at the end. It is also seen that on increasing the volume fraction of dust the initial velocity of the particle increases.

Thus we can conclude that the effect of volume fraction on the velocity of both the phases is same. It has been observed that the effect of magnetic field parameter is significant. The presence of magnetic field puts the system in wavy form with same amplitude till end, in case of fluid velocity, which is also evident mathematically as it contains sine and cosine terms. In case of particle velocity the graph also takes wavy form but with increasing amplitudes.

References


